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A METHOD OF INCORPORATING BEND FLEXIBILITY FACTORS IN MODELS FOR PIPING FLEXIBILITY ANALYSIS

PETER H. FREEMAN

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A METHOD OF INCORPORATING USER FLEXIBILITY ANALYSIS

IN MODELS FOR PIPING FLEXIBILITY ANALYSIS

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Peter L. Freeman

A METHOD OF INCORPORATING BEND FLEXIBILITY FACTORS IN MODELS FOR PIPING FLEXIBILITY ANALYSIS

by

Peter M. Freeman

Lieutenant, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
MECHANICAL ENGINEERING

United States Naval Postgraduate School
Monterey, California

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FREEMAN, P.

by

Peter H. Kreamer

This work is accepted as fulfilling
the thesis requirements for the degree of

MASTER OF SCIENCE

IN

MECHANICAL ENGINEERING

from the

United States Naval Postgraduate School

[illegible] flexibility [illegible] of tests [illegible] models is undertaken [illegible] simple and [illegible] to analytic or graphical methods of [illegible]. The difficulty [illegible] experimental-test methods is the problem of incorporating in current [illegible] the model the extra bending flexibility encountered in curved pipe.

This paper describes an attempt made to solve the problem by the use of very thin-walled tubing as a model in an already existing apparatus which permits determination of reactions by measurements of deflections in the model. Although the attempt was unsuccessful because the tubing was unknowingly loaded beyond its yield point, it is concluded that the basic idea does hold some promise for moderate values of bend flexibility.

The assistance and encouragement of Professor John E. Brock of the faculty of the U. S. Naval Postgraduate School, Monterey, California, is gratefully acknowledged.

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TABLE OF SYMBOLS

A	Extremity of pipe or model opposite the extremity at the origin of the stated problem.
d	Mean diameter of pipe or tube in inches.
d_i	Inside diameter of pipe or tube in inches.
d_o	Outside diameter of pipe, tube or rod in inches.
D_2	Translation in inches in y" direction of point P.
D_3	Translation in inches in z" direction of point P.
D_4	Angular displacement in radians about x" axis measured at P.
D_5	Angular displacement in radians about y" axis measured at P.
D_6	Angular displacement in radians about z" axis measured at P.
E	Modulus of elasticity in pounds per square inch.
F	Force in pounds acting on structure; see Table of Subscripts.
$\overline{F}$	Resultant force in pounds acting on structure.
F_2	Shear force in pounds at A in y' direction.
F_3	Shear force in pounds at A in z' direction.
F_4	Moment in pound-inches at A about x' axis.
F_5	Moment in pound-inches at A about y' axis.
F_6	Moment in pound-inches at A about z' axis.
J	$\pi(1 + \mu)r^2$ in inches^2
G	Modulus of elasticity in shear in pounds per square inch.
I	Moment of inertia of cross section with respect to neutral axis in inches^4.
J	Polar moment of inertia in inches^4.
k	Shape flexibility factor (See text, page 3). Dimensionless.
l_o	Free length of a spring in inches.
L	Linear distance in inches from A to P.

M	Moment in inch-pounds acting on structure; see Table of Subscripts.
$\overline{M}$	Resultant moment in inch-pounds acting on structure.
n	Number of turns in a spring.
O	Point of origin of problem as originally stated, and origin of the unprimed system of coordinates.
P	Point of measurement of deflections, found at longitudinal center of mirror-mounting barrel.
Q	First moment of area of half of cross-section with respect to neutral axis in inches^3.
r	Mean radius of pipe or tube in inches.
$\bar{r}$	Mean radius of a coil of a spring in inches.
r_i	Inside radius of pipe or tube in inches.
r_o	Outside radius of a pipe or tube in inches.
R	Radius of bend in inches.
s_n	Normal stress in pounds per square inch.
s_s	Shear stress in pounds per square inch.
t	Wall thickness in inches.
X,Y,Z	x, y, and z coordinates of A.
x',y',z'	Coordinates with origin at anchor with directions same as x", y" and z" respectively. See Fig. 4.
x,y,z	Coordinates of a general point taken with respect to origin at O as stated in original problem. See Fig. 4.
x'',y'',z''	Coordinates with origin at P. See Fig. 4.
x''',y''',z'''	See Appendix II.
α	Angle of inclination in radians of any coil of a spring with a circular section of the cylinder on which the coil lies.
β	Flexural rigidity in pounds-inches^2.
γ	Angle between a radius line of a cross-section and the horizontal, measured counter-clockwise in radians.

δ	Deflection due to shear in inches.
Δ_m/Δ_p	Ratio of deformations applied in model to prototype deformation, dimensionless.
θ	Slope of shear deflection curve in radians.
μ	Poisson's ratio, dimensionless.
ξ, η, ζ	Coordinates of a reflected point of light on cross section paper for rotation measurement, such that $\xi_2 - \xi_1, \eta_2 - \eta_1$, and $\zeta_2 - \zeta_1$ indicate angular displacements about x'', y'', and z'', respectively in millimeters.
T	Torsional rigidity in pounds-inches^2.

TABLE OF SYMBOLS

A	Refers to point A.
m	Refers to model.
O	Refers to point O.
p	Refers to prototype (actual piping system).
x, y, z,	Refer to components in directions x, y, and z, respectively.
x', y', z'	Refer to components in directions x', y', and z', respectively.
x", y", z"	Refer to components in directions x", y", and z", respectively.
1	Pertains to set of data taken with system deformed in negative direction. This does not apply to use of numbered subscripts with A or B.
2	Pertains to set of data taken with system deformed in positive direction. This does not apply to use of numbered subscripts with A or B.

1. Introduction

[illegible] the desirability of [illegible] for [illegible] the reactions of a variety of [illegible] systems subjected [illegible] [illegible] [illegible]

reduction in the moment of inertia. For bending moments in the plane of the bend the major axis of the ellipse is normal to the plane of the bend, and for out-of-plane bending it is inclined at some other angle. The resultant decrease in rigidity is the same, however, regardless of whether the bending is in- or out-of-plane. The effect appears to be independent of internal pressure, but is considerably modified by the presence of any flanges or stiffening rings [1] .

2. Purpose of this work

In his tests Rauch used models constructed of solid rod and chose to neglect bend flexibility factors or to analyze systems where the factors were unity. The purpose of the work described herein was to extend his method to include these effects. Thus a closer examination of bend flexibility is in order.

Various expressions for bend flexibility factors k have been proposed, but A. E. C. Markl [1] suggests that the following simplified form is entirely adequate for most purposes:

$$k = \frac{1.65\ r^2}{tR}\ , \text{ where:} \qquad (1)$$

r is the mean radius of the undeformed pipe,

t is the wall thickness, and

R is the radius of the bend.

This formula is incorporated in the code for pressure piping ASA 1331.1 - 1955.

In straight pipe the ratio of torsional to flexural rigidity is $\frac{1}{1+\mu}$, where μ is Poisson's ratio. In bends this ratio becomes $\frac{k}{1+\mu}$. The problem here, then, was to devise some means of reducing the flexural rigidity from EI to EI/k in the bends of a model while keeping the torsional rigidity constant throughout the model. At the same time it was desired that the method used be in keeping with the basic simplicity of Rauch's apparatus. Various ideas were considered before the one used was finally adopted. Thought was given to some scheme for merely modifying the cross-section of the bends in a solid rod model, but this would not work because a solid circular cross-section provides the maximum

ratio of torsional to flexural rigidity, and any change in the shape of cross section serves to decrease rather than to increase the ratio. However, it should be noted in passing that for single-plane configurations, where no torsion is involved, one should be able to produce increased flexibility in bends merely by reducing the moment of inertia of the rod by the factor k.

Another idea considered was based on an article by J. D. Conrad [3] in which he described a method of increasing bend flexibility by using fairly thin-walled tubing for models and cutting slots in the bends. Correspondence with Mr. Conrad, however, revealed that he had no analytical method of pre-determining the slot depth or distribution, but that, instead, a "cut and try" approach was used. Such a procedure did not appeal to the present investigator.

It was next thought that perhaps a coil spring might be inserted in the place of the rod in the bends. However, an analysis (Appendix III) based on Wahl's [6] and Prescott's [7] spring equations showed that the maximum flexibility factor that could be simulated in this way would be 1.50; it was decided that this gave a range too restricted for the present investigation.

Consideration was also given to the possibility of constructing a model along the lines of a simplified analytical method proposed by Markl [5], but it developed that of the three simplifying assumptions necessary to this method only one could be incorporated in a rod model.

Correspondence from Lale C. Andrews, formerly in charge of the M. W. Kellogg Company testing laboratory, states that he has invented a device for simulating bend flexibility but is loathe to divulge any details at the present time.

A practical and simple approach to the simulation of flexibility factors of moderate values appeared to lie in the use of tubing for models, and consideration of attention was restricted to problems involving such moderate values. Literature on the subject of model-testing and correspondence with other investigators indicated that tubing with sufficiently small wall thickness for the purpose was not readily available, but this turned out to be not strictly true. As will be seen later, the difficulty lay not in obtaining satisfactory tubing but in bending it. Let us see what would be required in a tube. Forces and moments at the ends of the model are to be calculated by measuring deformations at some point in the straight section at one end and using cantilever beam formulae. Scale factors relating forces and moments calculated in the model to those in the prototype are [9] :

$$\frac{F_p}{F_m} = \frac{E_p}{E_m} \frac{I_p}{I_m} \frac{\Delta_m}{\Delta_p} \frac{\lambda_m^3}{\lambda_p^3} \qquad (2)$$

$$\frac{M_p}{M_m} = \frac{E_p}{E_m} \frac{I_p}{I_m} \frac{\Delta_m}{\Delta_p} \frac{\lambda_m^2}{\lambda_p^2} = \frac{F_p}{F_m} \frac{\lambda_p}{\lambda_m} \qquad (3)$$

$\frac{\Delta_m}{\Delta_p}$ need only be large enough so that an adequate cantilever length, L, be provided for measurement of deflections and small enough so that the model will fit within the existing framework of the apparatus. These were the only limitations posed when rod was used for a model, but the use of tubing makes a further demand that the scale not produce model bend radii smaller than the practical limit for the tube size. Finally introduction of the requirement that the bend flexibility factors of model and prototype be the same still further restricts the choice of $\frac{\Delta_m}{\Delta_p}$ as follows:

$$\left(\frac{1.65r^2}{tR}\right)_m = \left(\frac{1.65r^2}{tR}\right)_p$$

$$\left(\frac{d^2}{t}\right)_m = \frac{Rm}{Rp}\left(\frac{d^2}{t}\right)_p$$

$$\left(\frac{d^2}{t}\right)_m = \frac{\lambda_m}{\lambda_p}\left(\frac{d^2}{t}\right)_p \qquad (4)$$

Thus it can be seen that the linear and cross-sectional scales are interdependent. Whether or not they can be equal to each other depends on the prototype dimensions and the dimensions of available tubing. However, other than for the esthetic value there is no particular reason to make them the same.

2. Apparatus

As mentioned previously, Gauch's apparatus, which is carefully described in the literature [8], was used for the tests conducted by the present investigator. Only a few modifications were found necessary to adapt it to the use of tubing instead of rod. The face plate of the milling head was removed, and in its place was installed an adaptor manufactured for this purpose. (Fig. 1). Two identical end-plates (Fig. 1) were manufactured to accept the ends of the tubing. Note that these plates could be used for any size tubing merely by boring the central hole to the proper diameter. A new anchor plate was manufactured, as was a mirror-mounting barrel of the proper size. (Fig. 2). Finally larger bakelite knife edge supports were cut out and installed.

4. Selection of Scale

This is necessarily a trial-and-error process. The problem selected for test is shown in Fig. 1. First it was decided that a minimum cantilever length, L, (between the anchor and the point of measurement, P) of eight inches would be acceptable. With end sections of around 120 inches this meant a maximum length scale of about 15:1. Inserting this value, together with the given dimensions of the pipe, into equation (4) gave a value of 44.6 for $\left(\frac{d^2}{t}\right)_m$. Examination of the catalogs of several manufacturers of tubing indicated that the smallest tubing with approximately this value of $\frac{d^2}{t}$ had an outside diameter of 1.25 inches and a wall thickness of .022 inch, for which $\frac{d^2}{t}$ is 56.2. Using 65.2 in the same equation, but this time solving for $\frac{\lambda_p}{\lambda_m}$ gave a scale of 9.498, which was entirely satisfactory.

5. Preparation of the Model

As mentioned previously, other investigators have remarked that suitable thin-wall tubing was not readily available. In the present investigation it was found that suitable tubing was available -- although a smaller diameter would have been more desirable -- but that the real difficulty lay in bending the tube into the required form for the model. From the beginning it was realized that bending without crimping would not be easy, and to allow for initial failures enough yellow brass tubing was ordered to make four models. This material was chosen for its ease of bending. (As described in paragraph 6, however, this material proved to have inadequate yield strength in the softened condition.) As it turned out, all the material was actually used. To support the tube during the bending operations it was filled with a low-melting alloy, "Fusible Alloy No. 158," a product of the American Smelting and Refining Company. It was thought that, thus supported, the tube could be bent with a ram type bender locally available. However, this proved far from true, for on almost the first touch of the ram the tube crimped. Eventually the three remaining lengths of tubing were taken to the San Francisco Naval Shipyard. The tubing was flame-annealed, and after two unsuccessful attempts one satisfactory model was obtained from a large compression-bending machine and ball-head mandrel but without the use of any filler. In addition to the unavoidable thinning of the walls on the outsides of the bends the resulting model actually had the following defects:

The portion between the two bends was distorted for about four inches as shown roughly in Fig. 5.

Instead of being exactly 90 degrees, one bend was 89 degrees and the other, 86 degrees.

Once the tube was bent, the mirror-mounting barrel was slipped over one end and the two end-plates silver-soldered in place.

6. Procedure

(a) Preparation of the Apparatus

In addition to simple rearrangement of the various components of Rauch's apparatus, it was found necessary to raise the compound feed device nine inches to accommodate the model to be used. This was accomplished by constructing a sturdy, table-like foundation and welding it in place on top of the existing foundation. (Fig. 3). The compound feed was then aligned and bolted in place.

The assemblies were roughly aligned and clamped in place. To adjust the deformation assembly relative to the anchor point in such a way that exactly the right X, Y, and Z dimensions were provided, the two end-plates were temporarily installed (subsequently to be removed and silver-soldered to the tube as described in paragraph 5). An eight-inch length of one-eighth inch diameter steel rod, threaded at one end and having a thin slot in the other end, was screwed into the hole provided for it in the center of the end-plate mounted on the anchor plate. A small, precision plumb bob was suspended from the end of the rod, supported by the friction of its thread in the slot. When adjusted so that it just touched the floor, the point of contact was marked and the height measured. Rod and bob were then removed and installed in the other end-plate (mounted on the deformation assembly), which could then be positioned to its correct height. To achieve the correct horizontal position, the deformation assembly was moved until the plumb bob touched a point marked on the floor by measurements from the point previously marked for the anchor.

(b) Installation of the Model

Upon proceeding to bolt the model in place by means of its end-

plates, it was found that the tube in the end-plate at one end and did not fit [illegible] in the end plate. The end-plate was, therefore, removed, [illegible] and the tube was again silver-soldered in [illegible].

During this experiment [illegible] deformation [illegible] was not [illegible] of how the [illegible] compared with the [illegible] -- to such an extent that any results would probably be [illegible]. Rather than to construct a whole new model, it was decided simply to cut off the deformed parts (about three-quarters of an inch), reinstall the model, and make the necessary slight changes in the initial problem dimensions. This was done without removing either end-plate from the apparatus.

Three sets of readings were then taken for each of the three deformations ratios (0.25, 0.50, 0.75). Since it was now known that the end-plates were not installed accurately enough to permit simple reversal of the tube without putting it under considerable initial stress, the model was to be removed from its end-plates (by melting the solder), reversed, and re-soldered. Unfortunately during this process a hole was burned through the tube at one end. This required cutting off the damaged portion (about one-half inch) and making another (final) alteration to the initial problem dimensions. The resulting dimensions are those shown in Fig. 4.

(c) Measurement of Deflections

As mentioned earlier, Rauch's system was used for measuring both translation and rotation. With the cantilever length, L, of 10.25 inches chosen, readings were being taken just short of the beginning of

the same clevis to the anchor point. Three deformation ratios were used -- 0.50, 1.00, and 1.50 -- in order to determine the effect any variation of this ratio might have on the results when a rubber model was used. Half the total deformation was applied in the negative direction, and readings of y'', z'', ξ, η, and ζ taken. Then the same deformations were applied in the positive direction and the same readings taken. This sequence was followed at least three times for each deformation ratio. First results were not encouraging; readings obtained with the same deformations were not repetitive. But as the procedure of shifting from negative to positive deformations was repeated again and again correlation improved markedly until finally, it was deemed satisfactory. It is believed by the writer that the improvement was the result of working out stresses locked in the tube by the heat applied at its ends during soldering.

The deformations thus measured were sufficient to lead to all reactions except the axial force F_x. To obtain the deformations necessary to calculate this force and to check the other reactions the tube was reversed and another set of readings taken using the same deformation ratios.

Three points of technique are considered worthy of mention. First, in the measurement of translation it was found essential to back off one knife edge a set, exact amount before taking a reading with the other knife edge. This procedure ensures that all readings are taken at the same point on each knife edge and removes the effects of any misalignment of the edges.

A second point is that in moving the translation-measuring device great care must be taken not to drive the knife edges hard against

[illegible] two sections selected are [illegible] and reactions at point [illegible] [illegible] ics.

7. Calculations

The deflections for all runs for each deformation ratio were averaged. Deflections were corrected using Rauch's calibration curves for the optical systems. Table 1 lists equations then used to solve for reactions. The equations for the rod were taken directly from Rauch's thesis. Those for the tube are also based on Rauch's, but because deflection due to shear was a factor to be considered with the tube, the equations were modified as shown. Appendix I gives the method of arriving at these modifications. Calculations of the various multiplication factors are shown in Table 2. Table 3 outlines a summary of deflections observed and calculations of reactions for the two models in both the normal and reversed positions. Tables 4 and 5 summarize results for the tube and the rod respectively. Values of F_1 are average of those obtained in the normal and the reversed positions. Table 6 includes the corresponding values of all reactions as obtained by Professor Brock's digital computer solution. Using the computed values as correct, per cent errors for all experimental values have been computed and listed in the two tables.

[illegible] small [illegible], observed [illegible] deformations in Table I are all too large.

It is obvious, rather, that the only part of the large errors [illegible] to consist [illegible] tube as and its elastic limit. It will be recalled that all [illegible] was used to secure [illegible] the tube and that [illegible] increases, while [illegible] temperature of 1000°F, was repeated several times. As the temperature [illegible] tube during soldering decreased rapidly [illegible] and [illegible] applied [illegible] the tube was subjected [illegible] series of [illegible]. Recall also that it had been [illegible]-annealed before bending. The yield point for yellow brass is approximately 11,000 psi [11] in the softened condition as against 57,000 psi in the hardened condition. An investigation (Appendix II) was made of the stresses at the anchored end of the tube in its normal position and under the smallest deformation ratio used, 0.20. The analysis showed that at one point the stress was 17,417 psi, which is well above the yield strength for the softened condition. For the higher deformation ratios used this figure would have been 34,834 and 52,251 psi.

If the tube had, in fact, been stressed beyond its yield strength it seemed logical that it should not return exactly to its no-load position in space when the load was removed -- that this position should change with each loading. Accordingly a check was made of the constancy of the position of the point of measurement P in the unloaded, or neutral, position. Because of the manner in which deformations had been applied -- that is, first in the negative direction and then in the positive and back again -- there had been no evident necessity for

taking readings on the neutral position. Therefore it was necessary to obtain an approximation to this position by assuming that it lay half way between those for the positive and the negative deformations. The readings y_1'' and y_2'', z_1'' and z_2'' etc. for each deformation ratio were averaged and the results assumed to be the position of P for each ratio. A very definite "drifting" of the point was indicated, bearing out the theory that inelastic behavior had occurred.

It is unfortunate that time did not permit re-running of the experiment with a new model of different material.

3. Conclusions

Despite the poor results obtained in the tests described herein, the investigator feels that the basic method of accounting for moderate bend flexibility factors by the use of tubular fan models is not without promise. Let us examine the difficulties and failures encountered and see how they might be overcome. First there is the model itself. It must be small enough in cross-section to be handled with a fair amount of ease, and it must be small enough in overall size to be accommodated within the existing apparatus. These two requirements combine to demand a thin-walled tube. Because of the difficulties attendant on bending such a tube, the work should be done by experienced personnel in a well-equipped shop; dies and mandrels must be of the exact size. Even under these conditions there will inevitably be a certain amount of thinning of the tube wall at the outside of bends and a corresponding thickening at the inside, but this should be similar to conditions in prototype pipe which has been bent. It should be noted too that there is a minimum limit to the radius which can be produced in any particular tube and thus to bends which can be simulated. These limits are tabulated in such books as reference 12, which also contains a great deal of other information on bending of pipe and tube. Welding bellows, for example, can probably not be simulated with tubing. In any case, the material selected for the model should have high ductility for ease of bending and a low modulus of elasticity to minimize the reaction on the apparatus. It should also be heat treatable in order to obtain uniform properties after installation of the end-plates. These end-plates should not have to be removed if properly installed initially, but mirror-mounting barrel should be made in

[illegible] to the other [illegible]. The writer suggests stainless steel as a [illegible], although the modulus of elasticity is high. It can easily be cast, can be heat-treated, and is available in an extremely wide variety of sizes with thin walls.

Supposing that a satisfactory model were obtained and mounted, there would still remain the possibility of stressing it beyond its elastic limit by applying deformations which were too large. (In the case investigated the maximum stress for deformation ratio 0.75 was extremely close to the yield point even for the hardened condition of the material.) Measurement or calculation of these stresses would be possible, of course, because the forces and moments causing them are the very reactions being sought by the tests. To reduce the danger of overstressing the model, then, it would seem advisable to use an extremely low initial deformation ratio, solve for the reactions, and, from them, for the maximum stress. The largest possible deformation ratio usable without exceeding the elastic limit could then be calculated by multiplying the initial deformation ratio used by the ratio of yield strength to the maximum stress developed in the initial test (applying some suitable safety factor). It might well be, however, that the maximum allowable deformation ratio would be so low that the initial deformations would be too small to be determined with necessary accuracy, but it is not believed that such would be true in most cases.

In summary the writer feels that the method of accounting for bend flexibility factors in the use of these models is one which should not be overlooked by persons interested in analyzing piping flexibility.

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BIBLIOGRAPHY

1. A. A. C. Wahl, Piping Flexibility Analysis, Transactions of the ASME, Vol. 77, No. 2, Feb. 1955.

2. E. Lundquist, Strength of Materials, 2nd Edition, Part II, McGraw-Hill, 1941.

3. J. A. Brock, A Matrix Method for Flexibility Analysis of Piping Systems, Journal of App. Mech., Vol. 19, No. 4, Dec. 1952.

4. J. A. Brock, Matrix Analysis of Piping Flexibility, Journal of Applied Mechanics, Vol. 22, No. 3, Sept. 1955.

5. A. A. C. Wahl, Simplified Analysis of Multi-Plane Systems, Piping Engineering (A publication of Tube Turns, Louisville, Ky.) Paper 4-05, July 1957.

6. A. M. Wahl, Mechanical Springs, Penton Publishing Co., 1944.

7. J. Prescott, Applied Elasticity, p. 275, Longman and Green, 1924.

8. C. F. Rauch, Jr., A Method of Piping Flexibility Analysis by Deflection Measurements in Scale Models, Thesis, U. S. Naval Postgraduate School, Monterey, California, 1957.

9. M. W. Kellogg Co., Design of Piping Systems, Second Edition, p. 200, John Wiley and Sons, 1956.

10. S. Timoshenko and G. H. MacCullough, Elements of Strength of Materials, Third Edition, Art. 62, D. Van Nostrand, 1949.

11. R. A. Wilkins and M. S. Bunn, Copper and Copper Base Alloys, p. 71, McGraw-Hill, 1943.

12. J. B. Schubert, Pipe and Tube Bending, Industrial Press, 1953.

13. J. F. Conrad, Model Tests Solve High Temperature Problems, Power, Vol. 84, No. 10, pp. 58-61, Oct. 1940.

APPENDIX I

MODIFICATION OF EQUATIONS RELATING DEFLECTIONS TO REACTIONS

Consider the portion of the tube between the point of measurement P and the anchor as a centilever beam under the influence only of a shear force F. The slope of the resulting deflection curve (10) is

$$\theta = \frac{dy}{dx} = \frac{S_{s\,max}}{G}$$

For vertical loading the maximum shear stress occurs in the horizontal plane of the neutral axis and is given to a good approximation by the formula

$$S_{s\,max} = \frac{FQ}{Ib}$$

In the case of a tube, b is 2t. Also

$$Q = 2\int_0^{\pi/2} (r \sin\gamma)\, tr\, d\gamma = 2r^2t$$

where γ is the angle between any radius line and the horizontal, measured counterclockwise. Finally

$$G = \frac{E}{2(1+\mu)}$$

Substituting into the slope equation gives

$$\theta = \frac{(F)(2r^2t)}{\frac{E}{2(1+\mu)}(I)(2t)} = \frac{2(1+\mu)r^2F}{EI}$$

Letting $\varepsilon = 2(1+\mu)r^2$, we have

$$\theta = \frac{\varepsilon F}{E I}$$

Integrating the slope from x = 0 to x = L, we arrive at the deflection at P due to shear,

$$\delta_s = \frac{\varepsilon FL}{EI}$$

In the experiments employing the tubular model, then, deflections read are the results of both bending and shear. Since Rauch's equations (Table 1) consider bending only, the deflections read must be decreased by that part of those deflections due to shear. That is,

$D_n = D'_n - D_{n,s}$, where

D_n is the deflection to be used in the equation;

D'_n is the corresponding deflection observed in the test;

$D_{n,s}$ is the corresponding part of the observed deflection which is due to shear.

Now let δ_2 and δ_3 be shear translations in the y" and z" directions and θ_4, θ_5, and θ_6 be rotations about the x", y" and z" axes, respectively, due to shear. Then for Equation 5 (Table 1):

$$F_2 = \frac{6EI}{L^2}\left[\frac{2}{L}\left(D_2 - \delta_2\right) + \left(D_6 - \delta_6\right)\right]$$

$$= \frac{6EI}{L^2}\left[\frac{2}{L}\left(D_2 - \frac{L}{EI}gF_2\right) + \left(D_6 + \frac{1}{EI}gF_2\right)\right]$$

$$= \frac{6EI}{L^2 + 6g}\left(\frac{2}{L}D_3 - D_5\right) \qquad (5')$$

Similarly for Equation 6:

$$F_3 = \frac{6EI}{L^2 + 6g}\left(\frac{2}{L}D_3 - D_5\right) \qquad (6')$$

Shear does not affect F_4; therefore Equation 7 is unaltered.

For Equation 8:

$$F_5 = \frac{2EI}{L^2}\left[\frac{L}{3}\left(D_5 - \theta_5\right) - \left(D_3 - \delta_3\right)\right]$$

$$= \frac{2EI}{L^2}\left[\frac{L}{3}\left(D_5 - \frac{1}{EI}gF_5\right) - \left(D_3 - \frac{1}{EI}gF_3\right)\right]$$

$$= -\frac{W_1}{L^2}\left(\frac{L}{3}\ D_1 - \ \right) - \ \frac{1}{g}\ \ \ \ (8')$$

Similarly, it can be shown that deflection 2 becomes:

$$y_2 = -\frac{1}{L^2}\left(\frac{L}{3}\ D_1 + D_2\right) + \ \frac{1}{g}\ \mathcal{S}\ F_2 \ \ \ \ (9')$$

It is assumed that shear deflection in the prototype is negligible because of its very large ratio of length to diameter. All customary methods of analysis make this assumption, although the effect can be incorporated in some analytical treatments.

APPENDIX II

ANALYSIS OF STRESSES AT POINT A

Consider the conditions at the anchored end of the tube (Point A) with the [illegible] ratio of 0.20 applied. By applying the proper scale factors to the [illegible] solution we find that the forces and moments acting on the tube are as follows:

$-F_x$ = -[illegible] $-M_x$ = -[illegible] in. lbs.

$-F_y$ = -[illegible] $-M_y$ = [illegible] in. lbs.

$-F_z$ = -[illegible] $-M_z$ = [illegible] in. lbs.

The [illegible] is concentric to M_y and M_z, which is

M_R = [illegible] in. lbs.

acting in the y' -z' plane as shown in Fig. 6 (b).

We now combine the resultant of F_y and F_z, which is

F_R = [illegible] lbs.,

acting in the y' -z' plane as shown in Fig. 6 (a).

It will now be seen that the axes of F_R and M_R are fortuitously close to normal to each other. For the sake of simplicity they will be considered exactly normal and designated z'' and z'' as shown in Fig. 6 (c). Let us now direct our attention at the point of intersection of the negative z'' axis with the outside surface of the tube. At this point the shear stress due to F_R is zero, and the normal stress due to F_R is

$$S_1 = \frac{M_R r_o}{I} = [\text{illegible}] \text{ psi.}$$

The stress due to F_x at the outside of the tube is everywhere

$$S_2 = \frac{F_x r_o}{J} = 2127 \text{ psi.}$$

The normal stress due to F_x is everywhere

$\sigma_F = \frac{F}{A} = -100$ psi, where A is the cross-section area of the tube.

The stresses calculated above are used to produce a Mohr circle (σ_1 , σ_2). This shows a maximum normal (principal) stress of 17417 psi. Since this is a linear system, the stresses at $\frac{\Delta\eta}{\Delta_T} = 0.50$ and 0.75 should be two and three times this amount, respectively.

The equations used herein are valid only for elastic behavior. Since the yield point of the brass used in the thin model was only 11,000 psi, it is evident that this point was exceeded, and thus the quantitative results of this analysis serve only to show that inelastic action must have occurred.

APPENDIX III

THE USE OF COIL SPRINGS TO SIMULATE PIPING FLEXIBILITY

It is desired to know whether coil springs could replace the bends of a solid rod model to simulate the bend flexibility behavior of a pipe. In piping the ratio of torsional rigidity (referred to herein as τ) to flexural rigidity in a bend (β) is:

$$\frac{\tau}{\beta} = \frac{k}{(1+\mu)}$$

Flexural rigidity of a spring with wire of circular cross-section is [8]:

$$\beta = \frac{l_0 EIG}{n \pi \bar{r}(2G+E)}$$, where:

l_0 is the free length of wire,

n is the total number of turns of wire,

$\bar{r}$ is the mean radius of the coil, and

I is the moment of inertia of the cross-section of the wire.

The corresponding torsional rigidity of such a spring is [9]:

$$\tau = \frac{1}{\frac{\cos^2\alpha}{EI} + \frac{\sin^2\alpha}{GJ}}$$

where α is the angle of inclination of the coils to a circular section of the cylinder on which the coil lies. E, I and J refer to the wire composing the spring.

To simplify the form of the ratio of the two rigidities, β and τ, we first reduce each to the terms of E, I, α, and μ as follows:

$$\frac{l_o}{n\pi r} = 2\,\frac{\frac{l_o}{2n}}{\pi r} = 2\sec\alpha$$

$$G = \frac{E}{2(1+\mu)}$$

$$J = 2I$$

$$\beta = \frac{2x2\sec\alpha\, EI\ \frac{E}{2(1+\mu)}}{2\,\frac{E}{2(1+\mu)} + E} = \frac{2EI\sec\alpha}{2+\mu}$$

$$T = \frac{1}{\frac{\cos^2\alpha}{EI} + \frac{\sin^2\alpha}{\frac{E}{2(1+\mu)}(2I)}} = \frac{EI}{\cos^2\alpha + (1+\mu)\sin^2\alpha}$$

$$\frac{T}{\beta} = \frac{EI}{\cos^2\alpha + (1+\mu)\sin^2\alpha} \times \frac{2EI\sec\alpha}{2+\mu} = \frac{(2+\mu)\cos\alpha}{2\left[\cos^2\alpha + (1+\mu)\sin^2\alpha\right]}$$

Equating this expression to the desired ratio and cross-multiplying we have

$$k = \frac{(1+\mu)(2+\mu)\cos\alpha}{2\left[\cos^2\alpha + (1+\mu)\sin^2\alpha\right]} \qquad (10)$$

For the case of a steel spring $\mu = 0.3$, and

$$k = \frac{2.99\cos\alpha}{2\,(\cos^2\alpha + 1.3\sin^2)}$$

Differentiating k with respect to α,

$$\frac{dk}{d\alpha} = \frac{-7.92\sin^3\alpha - 1.794\sin\alpha\cos^2\alpha}{4\,(\cos^2\alpha + 1.3\sin^2\alpha)^2}$$

It can be seen that within the conceivable range of α's for a coil spring (0 to $\frac{\pi}{2}$) k decreases with increasing α, so that the maximum value of k which can be realized would be with $\alpha = 0^\circ$. With such a spring, using equation (10),

$$k = \frac{2.99}{2} = 1.50.$$

TABLE 1

EQUATIONS RELATING OBSERVED DEFLECTIONS TO REACTIONS

REACTION	EQUATION NUMBER	RAUCH'S EQUATION[1]	EQUATION NUMBER	MODIFIED EQUATION[1]
F_2	5	$\frac{6EI}{L^2}\left(\frac{2}{L}D_2 + D_6\right)$	5'	$\frac{3EI}{L^2 + 6_g}\left(\frac{2}{L}D_2 + D_6\right)$
F_3	6	$\frac{6EI}{L^2}\left(\frac{2}{L}D_3 - D_5\right)$	6'	$\frac{6EI}{L^2 + 6_g}\left(\frac{2}{L}D_3 - D_5\right)$
F_4	7	$\frac{JG}{L}D_4$	7	$\frac{JG}{L}D_4$
F_5	8	$-\frac{6EI}{L^2}\left(\frac{L}{3}D_5 - D_3\right)$	8'	$-\frac{6EI}{L^2}\left(\frac{L}{3}D_5 - D_3\right) + 4\frac{L}{g}F_3$
F_6	9	$-\frac{6EI}{L^2}\left(\frac{L}{3}D_6 + D_2\right)$	9'	$-\frac{6EI}{L^2}\left(\frac{L}{3}D_6 + D_2\right) - 4\frac{L}{g}F_2$

[1] D_4, D_5, and $D_6 = (\xi_2 - \xi_1)$, $(\eta_2 - \eta_1)$ or $(\zeta_2 - \zeta_1)$, respectively times correction factor from optical system calibration curves (8) times 10^{-3}.

Calculations

(a) For [illegible]

$$\frac{2}{L} = \frac{L}{10.25} = 0.1951$$

$$\frac{I}{\quad} = \frac{[illegible]}{3} \quad [illegible]$$

$$\frac{6EI}{L^2} = \frac{6 \times [illegible] \times .016}{10.25^2} \quad [illegible] \times 10^4$$

$$\frac{JG}{L} = \frac{2 \times .016 \times 6 \times 10^6}{10.25} = 1.274 \times 10^4$$

$$\xi^2 = 2 \times (1+\mu) r^2 = 2 (1+0.3) 1.228^2 = 4$$

$$\frac{6EI}{L^2 \xi^2} = \frac{6 \times 15.7 \times 10^6 \times .016}{10.25^2 \times 4.0 \times 6} = 1173 \times 10^4$$

$$\frac{4}{L} = \frac{4}{[illegible]} = 1.561$$

$$\frac{F_p}{F_m} = \frac{E_m}{E_m} \frac{I_p}{I_m} \frac{\lambda_m^3}{\lambda_p^3} \frac{\Delta_m}{\Delta_p} = \frac{27.9 \times 10^6 \times 562.1 \times 1 \times \Delta_m}{15.7 \times 10^6 \times .016 \times 1.995^3 \times \Delta_p}$$

$$= 71.73 \frac{\Delta_m}{\Delta_p}$$

$$\frac{M_p}{M_m} = \frac{F_p}{F_m} \frac{L_p}{L_m} = 71.73 \frac{\Delta_m}{\Delta_p} \cdot [illegible] = .821 \frac{\Delta_m}{\Delta_p}$$

For $\frac{\Delta_m}{\Delta_p} = 2.[illegible]$, $\frac{F_p}{F_m} = 1.[illegible]$, $\frac{M_p}{M_m} = 1751.8$

For $\frac{\Delta_m}{\Delta_p} = [illegible]$, $\frac{F_p}{F_m} = 193.85$, $\frac{M_p}{M_m} = 1365.9$

For $\frac{\Delta_m}{\Delta_p} = [illegible]$, $\frac{F_p}{F_m} = 95.93$, $\frac{M_p}{M_m} = 910.6$

Table [illegible] Calculation of Multipliers

(b) For [illegible]

$$\frac{\Delta E I}{L} = -0.532$$

$$\frac{JG}{L} = 26.21$$

$$\frac{F_f}{F_n} = 50,900$$

$$\frac{M_f}{M_n} = 484,500$$

Table [illegible] of Natural [illegible]

Operation	Result	Sign	Col. →	1	2	3	4	5	6	7	8
			Δ_m/Δ_p →	.5	.5	.75	.25	.50	.75	1.0	1.0
$(-/)D_2$		+		.01012	.02109	.03418	.01169	.02345	.03504	.04706	.04646
$-D_6$		-		.00740	.01543	.02380	.00770	.01610	.02400	.03420	.03480
Alg. Sum.		+		.00272	.00566	.01038	.00399	.00735	.01104	.01286	.01166
$X6EI/(L^2+6g)$	F_{2m}	+		43.45	43.45	43.45	49.1	90.4	90.4	90.4	90.395
XF_p/F_m	$F_{y'p}$	+		9630	9630	12460	10130	13010	13010	13410	12210
$2/L\ D_3$		-		.03704	.07826	.03514	.03794	.01594	.02730	.03858	.03746
$-D_6$		+		.00510	.01040	.01580	.00570	.01170	.01800	.03300	.03200
Alg. Sum.		-		.00194	.00786	.00724	.00214	.00424	.00930	.00558	.00546
$X6EI/(L^2+6g)$	F_{3m}	-		33.85	19.78	89.2	263	52.1	1.2	.114	0.1122
XF_p/F_m	$F_{z'p}$	-		5860	8600	9530	7570	7500	10730	5840	5710
D_4		+		.00570	.01143	.01715	.00600	.01106	.01706	.02088	.01866
$(JG/E)XD_4$	F_{4m}	+		107.0	212.6	329.3	93.8	198.8	306.2	.5500	.5010
XM_p/M_m	$M_{x'p}$	+		232,000	289,400	294,300	258,200	271,400	291,500	266,000	242,000
$/3\ D_5$		-		.0174	.0338	.0483	.0198	.0340	.0480	.0128	.1093
$-D_3$		+		.0361	.0725	.1156	.0402	.0817	.1420	.1577	.1920
Alg. Sum.		+		.0187	.0387	.0673	.0204	.0417	.0740	.0849	.0827
$X(-6EI/L^2)$		-		272.0	527.3	838.3	301.3	606.0	1141	1.745	1.700
$(\ /\ )F_{3m}$		+		30.3	24.7	13.2	32.7	78.6	0	0	0
Alg. Sum.	F_{5m}	-		236.0	478.6	800.8	261.3	527.4	1141	1.745	1.700
XM_p/M_m	$M_{y'p}$	-		648,000	664,000	729,000	714,000	741,000	780,000	844,000	823,000
$L/3\ XD_6$		-		.0253	.0523	.0835	.0263	.0550	.0850	.1169	.1190
$+L_2$		+		.0519	.1091	.1712	.0658	.1292	.2007	.2811	.2881
Alg. Sum.		+		.0266	.0568	.0946	.0336	.0659	.1039	.1243	.1211
$X(-6EI/L_2)$		-		386.3	826	1380	480	949	1.745	2.490	2.490
$(\ /L\ X\ F_{2m}$		+		50.4	111	196	74	135	0	0	0
Alg. Sum	F_{6m}	-		336.1	715	1184	415	814	1.745	2.490	2.490
XM_p/M_m	$M_{z'p}$	-		910,000	978,000	1,058,000	1,134,000	1,112,000	1,270,000	1,236,000	1,203,000

NOTES: (1) This table constitutes a summary of the observed deflections and the calculations leading therefrom to the reactions at Point .
(2) The calculated reactions are those exerted by the pipe or tube on the structure.
(3) Columns 1 through 6 are for tests with the tubing model; columns 7 and 8, for the rod model.
(4) Odd-numbered columns are for the model in the normal position; even-numbered, for the reversed position.
(5) For the rod model, g is considered zero.

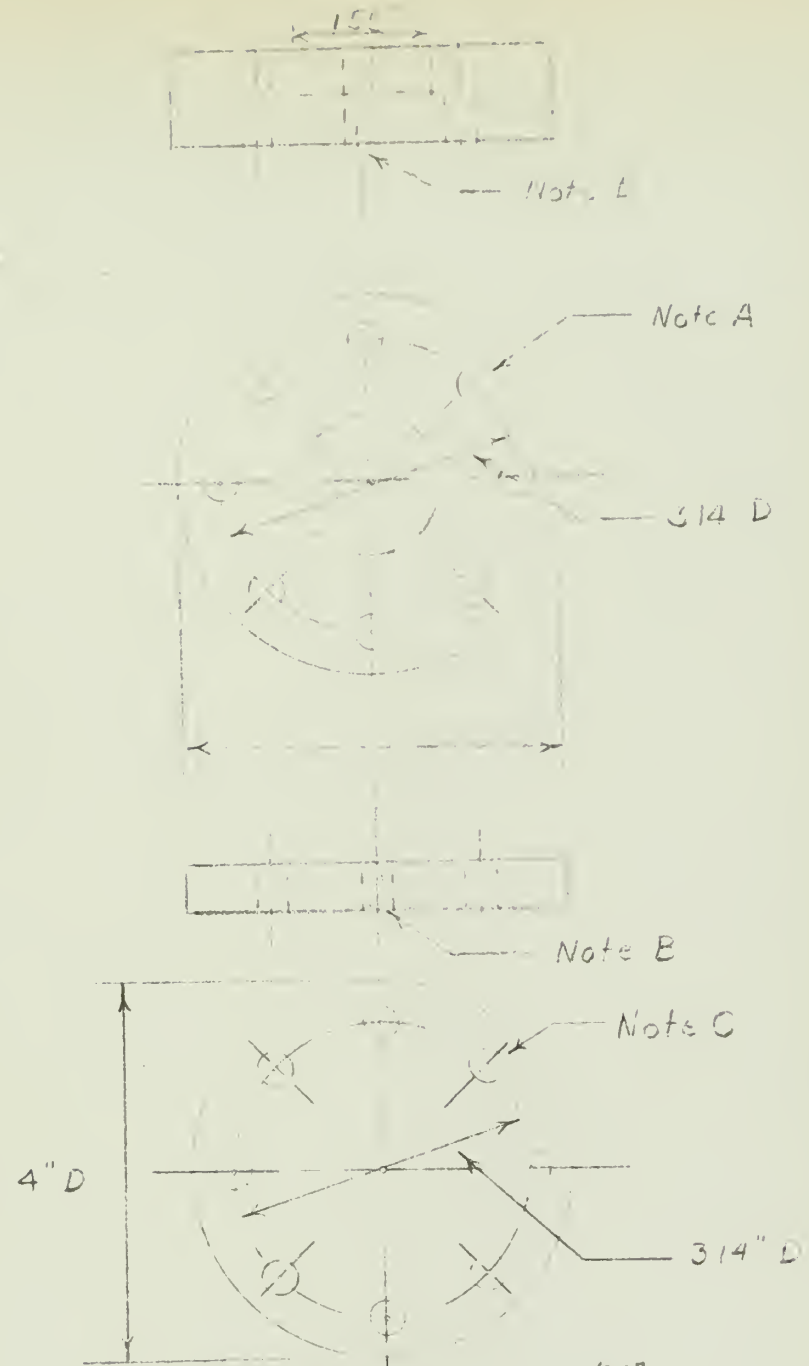
Table 3. Deflections Observed and Calculations made therefrom.

Mo-ment		Theoretical Value	$\frac{\Delta_z}{\Delta_p} = .2$	% Error	$\frac{\Delta_z}{\Delta_p} = .5$	% Error	$\frac{\Delta_n}{\Delta_p} = .75$	% Error
M_{xo}	-	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	10,730	124
F_{xo}	-	[illegible]	11,[illegible]	[illegible]	11,[illegible]	[illegible]	12,735	36.0
F_{zo}	+	4,[illegible]2	[illegible]	[illegible]	[illegible]	[illegible]	8,530	79.1
F_o		13,[illegible]3	13,[illegible]0	[illegible]	13,[illegible]0	[illegible]	13,650	61.9
M_{xo}	-	1,[illegible],205	1,134,000	6.5	1,112,000	4.4	1,270,000	18.2
M_{yo}	+	573,[illegible]60	714,000	23.5	721,000	24.7	885,000	53.2
M_{zo}	+	297,129	284,200	-13.8	271,400	-8.8	281,500	-5.4
M_o		1,248,000	1,301,000	10.3	1,354,000	8.6	1,593,000	27.7
F_{x_A}	+	4,781	7,570	58.0	7,100	67.5	10,730	124
F_{y_A}	+	9,365	11,230	20.9	11,805	26.1	12,735	36.0
F_{z_A}	-	4,762	6,800	44.1	8,600	80.7	8,530	79.1
F_A		11,533	13,640	18.7	16,370	42.1	18,650	61.9
M_{y_A}	+	296,000	292,000	-1.3	288,400	-2.4	299,000	1.0
M_{x_A}	-	577,000	645,000	11.8	654,000	13.3	729,000	26.4
M_{z_A}	-	1,0[illegible]1,000	919,000	-21.1	878,000	-8.7	1,078,000	0.7
M_A		1,250,000	1,155,000	-7.[illegible]	1,21[illegible],000	-3.2	1,335,000	6.8

Table 4. Summary of results using [illegible] model.

Parameter	Direction	Computer Value	Test Value	Percent Error
F_{xo}	→	7,382	[illegible]	-22.9
F_{yo}	↓	13,378	12,[illegible]30	- 4.1
F_{zo}	↓	[illegible],380	[illegible],840	-20.9
$\overline{F}_o$		14,968	15,1[illegible]0	-10.7
M_{xo}	→	1,239,370	1,203,000	- 2.9
M_{yo}	↓	903,803	831,[illegible]00	- 7.8
M_{zo}	↓	260,030	242,000	- 6.9
$\overline{M}_o$		1,[illegible],633	1,482,000	- 4.7
F_{xA}	↓	7,382	[illegible],710	-22.7
F_{yA}	↓	13,378	12,830	- 4.1
F_{zA}	—	[illegible],380	5,040	-20.9
$\overline{F}_A$		14,968	15,1[illegible]0	-10.7
M_{xA}	↓	263,000	266,000	- 1.4
M_{yA}	→	[illegible],000	833,500	- 8.0
M_{zA}	→	1,241,[illegible]	1,233,000	- 0.4
$\overline{M}_A$		1,[illegible],000	1,[illegible]12,000	- 3.0

Table 5. Summary of Results using Rod Model.



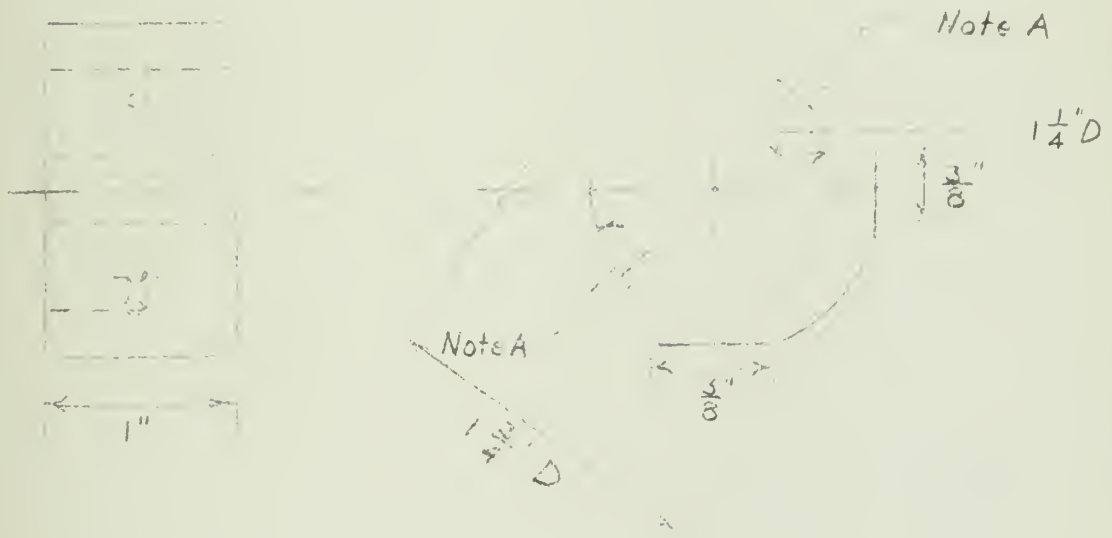
NOTES: A - Drill and tap all holes on .314" D for 5/16" cap screw.

B - Drill and tap for 1/2" NC thread.

C - Drill only for 5/16" cap screw.

D - Material: mild steel.

Figure 1. Milling head adaptor and end-plate.



NOTES: A - Drilled and tapped for 1/8 inch set screw.

B - Material is brass with outside surfaces silver-plated.

C - Front-silvered mirrors mounted on each 3/8 inch flat.

Figure 2. Mirror-Mounting Barrel

	PIPE	TUBE
b	164.77"	11.24"
c	45.41"	4.78"
e	106.49"	11.21"
R	80.00"	8.42"
d	15.625"	1.228"
t	0.375"	.022"
E	27.9×10^6 psi	15.9×10^6 psi
G	10.7×10^6 psi	6.0×10^6 psi
I	$562.1"^4$	$.0160"^4$
J	$1124.2"^4$	$.0320"^4$
k	3.357	3.357
L		10.25"
x	0.8069"	
y	0.3889"	
z	0.8120"	

Figure 4. Dimensions and Required Properties of Prototype System and Tuning Model.

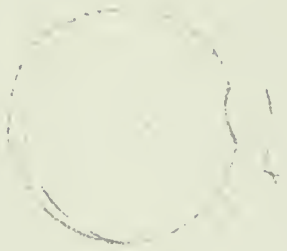


Figure 5. [illegible] Profile of Distorted Cross-Section Obtained by Use of Modelling Clay

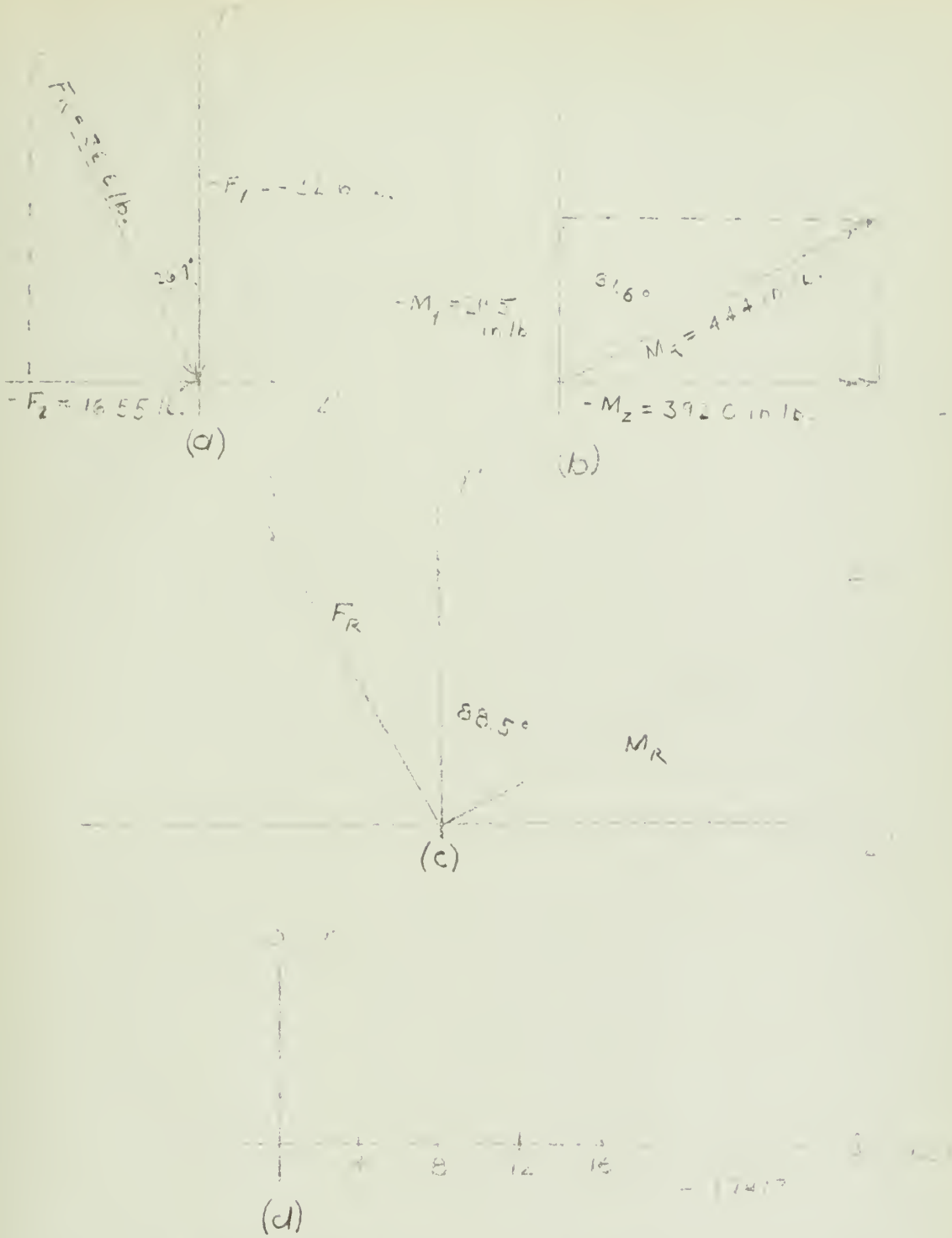


Figure 6. Graphical Summation of Forces, Moments and Stresses at Point A.

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